HOSILALI NOCHIZIQLI SHRODINGER TENGLAMASINI KELTIRIB CHIQARISH

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Kalit so‘zlar: Hosilali nochiziqli Shryodinger tenglamasi, tez kamayuvchi funksiyalar, Shvarts fazosi, integrallash usullari, matematik fizika tenglamalari, Fure almashtirishi, yechimning mavjudligi va yagonaligi.

Kirish. Zamonaviy matematik fizika va chiziqli bo‘lmagan differensial tenglamalar nazariyasida Shryodinger turidagi tenglamalarni tadqiq qilish muhim o‘rin tutadi. Xususan, hosilali Shryodinger tenglamalari kvant mexanikasi, plazmalar fizikasi, noldan farqli optik tolalar va to‘lqin jarayonlarini modellashtirishda keng qo‘llaniladi. Bunday tenglamalarning cheksiz sohalarda, xususan, tez kamayuvchi funksiyalar sinfida (Shvarts fazosida) yechimlarini o‘rganish nazariy va amaliy jihatdan katta qiziqish uyg‘otadi. Ushbu maqolada hosilali Shryodinger tenglamasini aynan shu sinfda integrallash masalalari, yechimlarning silliqligi va baholanishi chuqur tahlil qilinadi.

Keywords: Derivative Schrödinger equation, rapidly decreasing functions, Schwartz space, integration methods, mathematical physics equations, Fourier transform, existence and uniqueness of solutions.

Introduction: In modern mathematical physics and the theory of nonlinear differential equations, the study of Schrödinger-type equations plays a crucial role.

Specifically, derivative Schrödinger equations are widely used to model quantum mechanics, plasma physics, nonlinear optics, and various wave phenomena. Investigating the solutions of such equations in infinite domains, particularly within the class of rapidly decreasing functions (Schwartz space), is of significant theoretical and practical interest. This paper is devoted to the integration of the derivative Schrödinger equation within this specific function class, thoroughly analyzing the smoothness, behavior, and estimates of the solutions.

1967-yilda KdV tenglamasi va 1972-yilda Zaxarov-Shabat tomonidan NLS(Nonlinear Schrödinger Equation) tenglamasi yechilgach, matematiklar "Yana qaysi nochiziqli tenglamalarni shu usul bilan yechish mumkin?" degan savol ustida ish boshladilar. 1974-yilda Ablovitz, Kaup, Newell va Segur umumiy spektral strukturani taklif qilishdi. Biroq, bu klassik sistema nochiziqlikda hosila qatnashgan tenglamalarni (masalan, DNLS) qamrab olmas edi.

1970-yillarning boshiga kelib, matematiklar oddiy Nochiziqli Shredinger tenglamasi (NLS) ustida ishlayotgan edi. Biroq, plazma fizikasida, ayniqsa magnitlangan plazmada tarqaluvchi parallel Alven to'lqinlarini o'rganishda muammo paydo bo'ldi. Norvegiyalik fizik **Einar Mjølhus1** 1976-yilda o'zining mashhur ***"On the Modulational Instability of Hydromagnetic Waves Parallel to the Magnetic Field"*** maqolasida Alven to'lqinlarining tarqalishini tavsiflovchi tenglamani aniq shaklga keltirdi:

$$i\frac{\partial q}{\partial t}+\frac{\partial^2 q}{\partial x^2}+i\frac{\partial q}{\partial x}\left(|q|^2 q\right)=0 (*)$$

Bu tenglama **hosilali nochiziqli Shrodinger tenglamasini (DNLS) o‘zi edi.** Ushbu olimlarning ishlari solitonlar nazariyasida **Kaup-Nyuel** sistemasining paydo bo'lishiga zamin yaratdi.

1 **Einar Mjølhus** — norvegiyalik taniqli fizik va matematik, plazma fizikasi hamda nochiziqli to‘lqinlar nazariyasi bo‘yicha jahon darajasidagi mutaxassis. Uning ilmiy faoliyati, ayniqsa, **hosilali nochiziqli Shredinger tenglamasining (DNLS)** fizik asoslarini yaratishda poydevor bo‘lib xizmat qilgan.

Devid Kaup2 (David J. Kaup) va Alan Nyuel 3(Alan C. Newell) 1978-yilda ***"An exact solution for a derivative nonlinear Schrödinger equation"*** nomli ilmiy ishlarini e'lon qilishdi.

Kaup va Nyuel esa bu **hosilali nochiziqli Shrodinger tenglamasini (DNLS)** yechish uchun standart Zaxarov-Shabat sistemasini o'zgartirish kerakligini tushunib yetishdi. Ular spektral parametr ξ potensiallar bilan chiziqli emas, balki ko'paytma ko'rinishida ξq va kvadratik ko'rinishda ξ^2 qatnashadigan quyidagi

$$\begin{cases} v_{1x} = -i\xi^2 v_1 + q\xi v_2 \\ v_{2x} = i\xi^2 v_2 + r\xi v_1 \end{cases} (**)$$

Kaup-Nyuel sistemasini taklif qilishdi.

Ushbu Kaup-Nyuel sistemasini qaraymiz:

$$\begin{cases} v_{1x} = -i\xi^2 v_1 + q\xi v_2 \\ v_{2x} = i\xi^2 v_2 + r\xi v_1 \end{cases} (1)$$

Bu sistemani

shaklda yozish mumkin, bunda: $V = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$, $\delta_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ va $Q = \begin{pmatrix} 0 & q \\ r & 0 \end{pmatrix}$

Ikkinchi tomondan quyidagi

$$\begin{cases} iv_{1t} = Av_1 + Bv_2 \\ iv_{2t} = Cv_1 - Av_2 \end{cases} (2)$$

tenglikdan A, B, C nomalumlarni topish uchun $v_{1tx} = v_{1xt}$ va $v_{2tx} = v_{2xt}$ shartlardan foydalanamiz:

va

endi $v_{1xt} = v_{1tx}$ dan va $v_{2xt} = v_{2tx}$ dan quyidagilar

kelib chiqadi (3) ni soddalashtirib

2 **Devid J. Kaup** (David J. Kaup) — kelib chiqishi **amerikalik** bo‘lgan taniqli matematik va fizik. U Teskari sochilish metodi (TSM) va solitonlar nazariyasini rivojlantirishga ulkan hissa qo‘shgan olim hisoblanadi.

3 **Alan Nyuel** (Alan C. Newell) — **Irlandiyalik** matematik. U solitonlar nazariyasi, nochiziqli optika, turmush tarzi va tabiatdagi tartibsiz tizimlar (pattern formation) bo'yicha dunyoga mashhur mutaxassis.

$$\begin{cases} A_x = \xi Cq - B\xi r \\ iB_x = 2\xi^2 B - \xi q_t - 2i\xi qA \\ iC_x = 2i\xi rA - 2C\xi^2 - \xi r_t \end{cases} (3)$$

hosil bo'ladi. Endi A, B, C larni topish uchun

$n = 4$ da

$$\begin{cases} A = A_0 + A_1 k + A_2 k^2 + A_3 k^3 + A_4 k^4 \\ C = C_0 + C_1 k + C_2 k^2 + C_3 k^3 + C_4 k^4 \\ B = B_0 + B_1 k + B_2 k^2 + B_3 k^3 + B_4 k^4 \end{cases} (4)$$

(4) tengliklarni (3) ga qo'yib quyidagilarni topamiz:

Bu tengliklardagi k o'zgaruvchining darajalari oldidagi koeffitsientlarni tenglab olamiz:

$$k^5 : \begin{cases} qC_4 - rB_4 = 0 \\ 2B_3 - 2iqA_4 = 0 \\ 2irA_4 - 2C_3 = 0 \end{cases} \Rightarrow \begin{cases} B_3 = iqA_4 \\ C_3 = irA_4 \end{cases} (5)$$

$$k^4 : \begin{cases} A_{4x} = qC_3 - rB_3 \\ iB_{4x} = 2B_2 - 2iqA_3 \\ iC_{4x} = 2irA_3 - 2C_2 \end{cases} (6)$$

$$k^3 : \begin{cases} A_{3x} = qC_2 - rB_2 \\ iB_{3x} = 2B_1 - 2iqA_2 \\ iC_{3x} = 2irA_2 - 2C_1 \end{cases} (7)$$

$$k^2 : \begin{cases} A_{2x} = qC_1 - rB_1 \\ iB_{3x} = 2B_1 - 2iqA_2 \\ iC_{3x} = 2irA_1 - 2C_0 \end{cases} (8)$$

$$k^1 : \begin{cases} A_{1x} = qC_0 - rB_0 \\ iB_{1x} = -q_t - 2iqA_0 \\ iC_{1x} = 2irA_0 - r_t \end{cases} (9)$$

$$k_0 : \begin{cases} A_{0x} = 0 \\ iB_{0x} = 0 \,(10) \\ iC_{0x} = 0 \end{cases}$$

(1.5) dan

(1.10) dan $A_0 = B_0 = C_0 = 0$ desak

(1.9) dan

(7) dan $B_2 = C_2 = 0$

(3) dan $A_3 = 0$

(1) dan

kelib chiqadi.

(2) dan

(3) dan

(4) dan

Bundan

sistema hosil bo‘ladi. Bu sistemani birinchisini r ga, ikkinchisini q ga ko‘paytirib qo‘shsak,

A_2 ni topamiz:

va B_1, C_1 larni topamiz:

Demak

$$A = 2\xi^4 + \xi^2 rq$$
$$B = 2iq\xi^3 - q_x\xi + irq^2\xi \,(11)$$
$$C = 2ir\xi^3 + r_x\xi + ir^2q\xi$$

Endi (11) ni (3) ga qo‘yamiz:

Demak bunda $r = \pm q^*$ desak, ushbu

hosilali nochiziqli Shrodinger tenglamasini hosil qilamiz.

Foydalanilgan adabiyotlar

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